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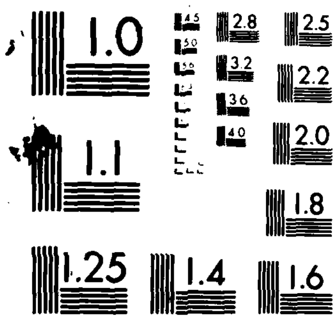

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In-House Report
November 1981

SECTOR SIDELOBE NULLING

Robert A. Shore
Ronald L. Fante

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The growing interest in adaptive antennas makes it worthwhile to examine the theoretical limits to what can be achieved with reduction of sidelobe levels of array antenna patterns. In this paper we address the probl[illegible] of determining the minimum possible power that can be present in a [illegible] sidelobe region subject to certain constraints. We consider three [illegible] forms of the minimization problem: 1) minimize the ratio of the pow[illegible] the sidelobe region to the power in the mainbeam region; 2) minimize the rat[illegible] of the power in the sidelobe region to the power density in the mainbeam direction; and

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20. Abstract (Continued)

3) minimize the power in the specified sidelobe region subject to the constraint that the array response in the mainbeam direction is held constant. It is shown that forms 2 and 3 of the minimization problem lead to identical results, and that the results obtained with form 1 are numerically close to those obtained with forms 2 and 3. We conclude that if all except one degrees of freedom of the element weights are utilized to minimize pattern power in a specified sidelobe sector, with the remaining one degree of freedom used to satisfy a mainbeam constraint condition, then a pattern can be formed with low average sidelobe power in a broad sector. The larger the number of array elements, the lower will be the average power in the specified sidelobe sector. The price paid, however, for this sector sidelobe power reduction is a marked degradation of the remainder of the pattern.

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1. INTRODUCTION

The growing interest in adaptive antennas makes it worthwhile to examine the theoretical limits to what can be achieved with reduction of sidelobe levels of array antenna patterns. In this report we address the problem of determining the minimum possible power that can be present in a specified sidelobe region subject to certain constraints. We consider three different forms of the minimization problem:

1) minimize the ratio of the power in the specified sidelobe region to the power in the main beam region;

2) minimize the ratio of the power in the specified sidelobe region to the power in the main beam direction; and

3) minimize the power in the specified sidelobe region subject to the constraint that the array response in the main beam direction is held constant.

When we refer to the main beam region or main beam direction we are referring to a pattern prior to the indicated minimization since the main beam, in general, will shift or transform in shape as a result of the change in array weights.

It is shown that forms 2 and 3 of the minimization problem lead to identical results, and that the results obtained with form 1 are numerically close to those obtained with forms 2 and 3.

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2. ANALYSIS

For all three forms of the sidelobe power minimization problem we consider an equispaced linear antenna array consisting of 2N + 1 isotropic elements spaced a distance d from each other (see Figure 1). The field radiation pattern of the array is

$$F(u) = \sum_{n=-N}^{N} a_n e^{-jnu}$$

where

a_n = the n^{th} complex array element weight

and

$u = kd \sin \theta,$

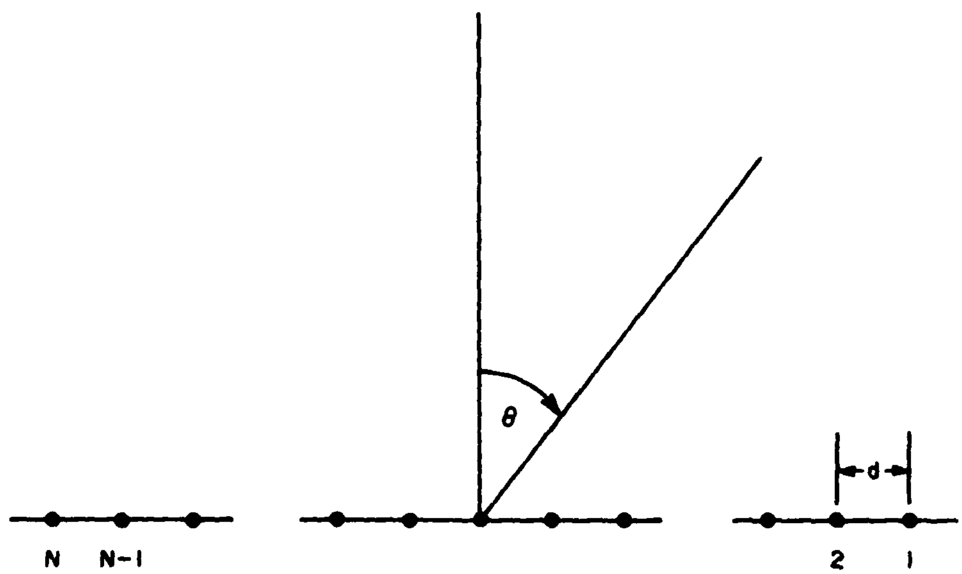

Figure 1. Geometry of Linear Array

with

$$k = \frac{2\pi}{\lambda}$$

and

θ = the pattern angle measured from broadside.

The corresponding power pattern is then given by

$$|F(u)|^2 = \sum_{n=-N}^{N} \sum_{n=-N}^{N} a_m^* a_n e^{-j(n-m)u} \quad . \tag{1}$$

The direction of the main beam prior to adjusting the array coefficients to reduce the power in a specified sidelobe region will be assumed throughout to be the broadside direction, $u = 0$, but the analysis presented here can be readily extended to cover any specified main beam direction.

2.1 Minimize the Ratio of the Power in the Specified Sidelobe Region to the Power in the Main Beam Region

The array pattern corresponding to uniform amplitude and zero phase element coefficients is

$$\frac{1}{2N+1} \quad \frac{\sin\left(\frac{2N+1}{2}\right)u}{\sin\frac{u}{2}} \quad .$$

We take the main beam region to be half the interval between the first nulls on either side of broadside, or

$$-u_B \leq u \leq u_B$$

where

$$u_B = \frac{\pi}{2N+1} \quad .$$

The power in the main beam region, P_B, is then given by

$$P_B = \frac{1}{kd} \int_{-u_B}^{u_B} |F(u)|^2 \, du \quad .$$

Substituting Eq. (1) for $|F(u)|^2$ and performing the integration then gives

$$P_B = \frac{2}{kd} \sum_{m=-N}^{N} \sum_{n=-N}^{N} a_m^* \, a_n \, \frac{\sin[(n-m)u_B]}{(n-m)}$$

$$= \frac{2\,u_B}{kd} \sum_{m=-N}^{N} \sum_{n=-N}^{N} a_m^* \, a_n \; \mathrm{sinc}\,[(n-m)\,u_B] \quad . \tag{2}$$

The sidelobe region is specified by

$$u_o - \varepsilon \le u \le u_o + \varepsilon$$

where

$$u_o = kd \sin\theta_o \quad .$$

The power in the sidelobe region is then given by

$$P_S = \frac{1}{kd} \int_{u_o-\varepsilon}^{u_o+\varepsilon} |F(u)|^2 \, du$$

$$= \frac{1}{kd} \sum_{m=-N}^{N} \sum_{n=-N}^{N} a_m^* \, a_n \int_{u_o-\varepsilon}^{u_o+\varepsilon} e^{-j(n-m)\,u}$$

$$= \frac{2\varepsilon}{kd} \sum_{m=-N}^{N} \sum_{n=-N}^{N} a_m^* \, a_n \, e^{j(n-m)\,u_o} \; \mathrm{sinc}\,[(n-m)\varepsilon] \quad . \tag{3}$$

We now desire to minimize the ratio of the power P_S in the sidelobe region to the power P_B in the main beam region. Thus we must minimize the quantity $\mu = P_S/P_B$, or substituting from Eqs. (2) and (3),

$$\mu = r \frac{\sum_{m=-N}^{N} \sum_{n=-N}^{N} a_m^* a_n H_{mn}}{\sum_{m=-N}^{N} \sum_{n=-N}^{N} a_m^* a_n G_{mn}} \tag{4}$$

where

$$r = \frac{\varepsilon}{u_B} \tag{5}$$

$$u_B = \frac{\pi}{2N+1}$$

$$H_{mn} = e^{-j(n-m)\,u_o} \operatorname{sinc}[(n-m)\,\varepsilon] \tag{6}$$

and

$$G_{mn} = \operatorname{sinc}[(n-m)\,u_B] \quad .$$

Equation (4) for μ can be written in the form

$$\mu(\underline{a}) = r \frac{\underline{a}^+ H \underline{a}}{\underline{a}^+ G \underline{a}} \tag{7}$$

where $\underline{a}$ is the vector of complex array element coefficients

$$a = \begin{bmatrix} a_{-N} \\ a_{-(N-1)} \\ \cdot \\ \cdot \\ \cdot \\ a_{N-1} \\ a_N \end{bmatrix}$$

and $\underline{a}^+$ is the Hermitian transpose of $\underline{a}$.

For μ to be a minimum it is necessary for μ to be stationary with respect to small variations in $\underline{a}$; that is, $\mu(\underline{a} + \delta\underline{a}) = \mu(\underline{a})$ to the first order for any small variation $\delta\underline{a}$. Equating $\mu(\underline{a} + \delta\underline{a}) = \mu(\underline{a})$ in Eq. (7) leads to the eigenvalue equation[1]

$$H\underline{a} = \frac{\mu}{r} G \underline{a}$$

or

$$H\underline{a} = \mu' G\underline{a}, \tag{8}$$

letting $\mu' = \mu/r$.

Now both H and G are Hermitian matrices and G is positive definite since a^+Ga equals the power in a region of the array pattern and so, is greater than zero for any set of array coefficients not identically equal to zero. Hence[2] the eigenvalue equation Eq. (8) has 2N + 1 linearly independent eigenvectors $\underline{a}_i$; $i = 1, 2, \ldots, 2N + 1$, satisfying the orthogonality relation

$$\begin{aligned} \underline{a}_i^+ G \underline{a}_j &= 1, \qquad i = j \\ &= 0, \qquad i \neq j \ . \end{aligned}$$

The smallest eigenvalue of Eq. (8) when multiplied by r as defined by Eq. (5) is the desired minimum value of the ratio of the power in the sidelobe region to the power in the main beam region.

2.2 Minimize the Ratio of the Power in a Specified Sidelobe Region to the Power Density in the Direction of the Peak of the Main Beam

This minimization is obviously equivalent to maximizing the ratio of the power density in the main beam direction (that is, broadside) to the power in the specified sidelobe region. Now the power density in the direction $u = 0$ is given by

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1. Pease, M.C., III (1965) Methods of Linear Algebra, Academic Press, New York, p. 91.
2. Gantmacher, F. (1960) The Theory of Matrices, Vol. 1, Chelsea Publishing Co., New York, p. 338.

$$|F(0)|^2 = \sum_{m=-N}^{N} \sum_{n=-N}^{N} a_m^* a_n$$

$$= \underline{a}^+ A \underline{a}$$

where $\underline{a}$ is the vector of element coefficients and

$$A = \begin{bmatrix} 1 & 1 & . & . & . & 1 \\ 1 & . & . & . & . & 1 \\ 1 & 1 & . & . & . & 1 \end{bmatrix} \tag{9}$$

$$= \begin{bmatrix} 1 \\ 1 \\ . \\ . \\ . \\ 1 \end{bmatrix} [1, 1 \ldots 1] \quad .$$

As in the analysis of the first form of the minimization problem, the power in the sidelobe region is given by

$$P_S = \frac{2\varepsilon}{kd} \sum_{m=-N}^{N} \sum_{n=-N}^{N} a_m^* H_{mn} a_n$$

$$= \frac{2\varepsilon}{kd} \underline{a}^+ H \underline{a} \quad , \tag{10}$$

with the matrix H defined by Eq. (6). Hence we wish to maximize the ratio

$$\mu = \frac{|F(0)|^2}{P_S}$$

$$= \frac{\underline{a}^+ A \underline{a}}{\frac{2\varepsilon}{kd} \underline{a}^+ H \underline{a}} \quad .$$

As above we are again led to an eigenvalue problem; namely,

$$A\underline{a} = \frac{2\varepsilon}{kd} \mu H \underline{a} \quad .$$

In this case, however, because of the fact that A is given by Eq. (9) there is only one non-zero eigenvalue[3] given by

$$\mu = \frac{1}{\left(\frac{2\varepsilon}{kd}\right)} \begin{bmatrix} 1, 1, \ldots, 1 \end{bmatrix} H^{-1} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{11}$$

with a corresponding eigenvector

$$\underline{a} = \frac{1}{\left(\frac{2\varepsilon}{kd}\right)} H^{-1} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad . \tag{12}$$

Note the μ equals the sum of all the elements of H^{-1} divided by $\frac{2\varepsilon}{kd}$ and that the array coefficients are the sums of the respective rows of H^{-1} divided by $\frac{2\varepsilon}{kd}$.

2.3 Minimize the Power in the Specified Sidelobe Region Subject to the Constraint that the Value of the Array Pattern in the Direction of the Peak of the Main Beam is Held Constant

The field radiation pattern of the array in the direction u = 0 is given by

$$F(0) = \sum_{n=-N}^{N} a_n \quad ,$$

so that the constraint that F(0) should remain constant is equivalent to requiring that

$$\underline{c}^{+} \underline{a} = F(0) = \text{constant} \tag{13}$$

3. Cheng, D.K. and Tseng, F.I. (1965) Gain optimization for arbitrary antenna arrays, IEEE Trans. Ant. Prop., Vol. AP-13, pp. 973-974.

where

$$\underline{c} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} .$$

Since the power in the sidelobe region is given by Eq. (10),

$$P_S = \frac{2\varepsilon}{kd} \underline{a}^+ H \underline{a}$$

the problem here is to determine the vector $\underline{a}$ of array element coefficients that minimizes $\underline{a}^+ H \underline{a}$ subject to the constraint Eq. (13). The solution of the constrained minimization problem is given by[4]

$$\underline{a} = \frac{H^{-1}\underline{c}}{\underline{c}^+ H^{-1} \underline{c}} F(0) \quad . \tag{14}$$

The denominator of this expression is the sum of the elements of H^{-1} and the i^{th} element of $\underline{a}$ is the sum of the elements of the i^{th} row of H^{-1} divided by the sum of all the elements of H^{-1}. Hence the sum of the element coefficients is F(0) as desired.

The minimum sidelobe region power is given by

$$\begin{aligned} P_S &= \frac{2\varepsilon}{kd} \underline{a}^+ H \underline{a} \\ &= \frac{2\varepsilon}{kd} \frac{[H^{-1}\underline{c}\, F(0)]^+ H [H^{-1}\underline{c}\, F(0)]}{(\underline{c}^+ H^{-1} \underline{c})^2} \\ &= \frac{2\varepsilon}{kd} \frac{|F(0)|^2 \underline{c}^+ H^{-1} \underline{c}}{(\underline{c}^+ H^{-1} \underline{c})^2} \\ &= \frac{2\varepsilon}{kd} \frac{|F(0)|^2}{\underline{c}^+ H^{-1} \underline{c}} \end{aligned}$$

4. Frost, O. L., III (1970) Adaptive Least Squares Optimization Subject to Linear Equality Constraints, SEL-70-055, Tech. Rep. No. 6796-2, Information Systems Laboratory, Stanford University, California, p. 6.

where, we have made use of the fact that H and hence H^{-1} is Hermitian to set $(H^{-1})^{+} = H^{-1}$. For this value of the minimum sidelobe region power, the ratio of the power density at u = 0 to the sidelobe region power is

$$\frac{|F(0)|^2}{P_S} = \frac{\underline{c}^{+} H^{-1} \underline{c}}{\left(\frac{2\varepsilon}{kd}\right)} .$$

Note that this ratio is exactly the value found above, Eq. (11), in the analysis of the second form of the minimization problem, for the maximum ratio μ of the power density at u = 0 to the power in the specified sidelobe region. Hence the minimum sidelobe region power obtainable with array coefficients that satisfy the constraint Eq. (13) is the same sidelobe region power that minimizes the ratio of sidelobe power to the power density at u = 0.

Comparing the expression for the array element coefficient vector $\underline{a}$ found in Eq. (14) with that found in the second form of the minimization problem, Eq. (12), we see that the two expressions agree to within a multiplicative constant. This is effectively equality since eigenvectors are determined only to within an arbitrary multiplicative factor. Thus forms 2 and 3 of the minimization problem lead to identical solutions.

3. NUMERICAL RESULTS AND DISCUSSION

FORTRAN computer programs were written and run on a CDC 6600 to calculate the element weights and corresponding power ratios and power patterns for forms 1 and 2 of the minimization problem. The inputs were the number of elements in the array, 2N + 1, the element spacing, d, in terms of the wavelength (that is, half wavelength spacing is given by d = 1/2), the center of the sidelobe region in degrees, θ_o, and the upper half width of the sidelobe region in degrees, $\Delta\theta$. In terms of the inputs the sidelobe region in "u space" is given by

$$u_o - \varepsilon \leq u \leq u_o + \varepsilon \qquad (15)$$

where

$$u_o = 2\pi d \sin \theta_o$$

and

$$\varepsilon = 2\pi d \, [\sin(\theta_o + \Delta\theta) - \sin\theta_o] \quad .$$

Note that the sidelobe region defined by Eq. (15) is symmetrical with respect to the center of the region in "u space" but not in "θ space".

The solution of the generalized eigenvalue Eq. (8) involved in the first form of the minimization problem was solved using the IMSL program EIGZC and the inversion of the matrix H in Eq. (12) in the second form of the minimization problem was performed using the IMSL program LEQ2C. For all calculations the sidelobe center θ_o taken equal to 35° and the element spading d was set equal to 1/2. For N = 3 (7 elements) calculations were performed for $\Delta\theta$ = 15°, 25°, 35°, 45°, and 55°; and for N = 4 and 5 (9 and 11 elements, respectively) calculations were performed for $\Delta\theta$ = 45° and 55°.

Table 1 gives the lower and upper limits of the sidelobe region in degrees, θ_- and θ_+, for the various values of $\Delta\theta$ used.

Table 1. Lower (θ_-) and Upper (θ_+) Limits of the Sidelobe Region Corresponding to a Sidelobe Region Center of 35° and an Upper Half Width of $\Delta\theta$

(o) $\Delta\theta$	(o) θ_-	(o) θ_+
15	22.40	50
25	16.33	60
35	11.97	70
45	9.34	80
55	8.46	90

Table 2 gives the ratio of sidelobe power to main beam region power, P_S/P_B, for the first form of the minimization problem, and the ratio of the sidelobe power to the power in the broadside direction, $P_S/|F(0)|^2$, for the second form of the minimization problem. For comparison purposes the same quantitities are also tabulated for a uniformly weighted array. Note that as the number of elements increases the sidelobe power decreases, since more nulls are available to be placed in the sidelobe region.

Table 2. Ratio of Sidelobe Power to Main Beam Region Power, P_S/P_B, for the First Form of the Minimization Problem, and Ratio of Sidelobe Power to Broadside Direction Power, $P_S/|F(0)|^2$, for the Second Form of the Minimization Problem. Also shown for comparison are the same quantities for a uniform array. Numbers in parentheses are exponents of 10

Seven Elements					
$\Delta\theta^{(o)}$	15	25	35	45	55
P_S/P_B	0.402(-8)	0.117(-5)	0.277(-4)	0.151(-3)	0.259(-3)
uniform	0.397(-1)	0.515(-1)	0.659(-1)	0.111(0)	0.136(0)
$P_S/\lvert F(0)\rvert^2$	0.192(-8)	0.606(-6)	0.159(-4)	0.949(-4)	0.169(-3)
uniform	0.881(-2)	0.114(-1)	0.146(-1)	0.246(-1)	0.301(-1)

Nine Elements		
$\Delta\theta^{(o)}$	45	55
P_S/P_B	0.795(-5)	0.164(-4)
uniform	0.707(-1)	0.843(-1)
$P_S/\lvert F(0)\rvert^2$	0.464(-5)	0.101(-4)
uniform	0.122(-1)	0.145(-1)

Eleven Elements		
$\Delta\theta^{(o)}$	45	55
P_S/P_B	0.416(-6)	0.104(-5)
uniform	0.607(-1)	0.648(-1)
$P_S/\lvert F(0)\rvert^2$	0.225(-6)	0.597(-6)
uniform	0.855(-2)	0.913(-2)

The element weights for the first and second forms of the minimization problem were found to be surprisingly close to one another, differing by, at most, about four percent for the cases studied. The power patterns, when plotted for corresponding cases of the two forms of the minimization problem, are virtually indistinguishable from each other. In Figure 2(a-e) we show the seven element patterns obtained for the second form of the minimization problem, in Figure 3(a and b) we show the nine element patterns, and in Figure 4(a and b) the eleven element patterns. For reference purpose the uniform-weighted seven, nine, and eleven element patterns are shown in Figures 5a, 5b, and 5c. All patterns are normalized to have the value of unity in the broadside direction, $\theta = 0$.

The basic features of all the minimization solution patterns are a placing of all available pattern nulls in the specified sidelobe region and a corresponding degradation of the remainder of the pattern. For a given number of elements, the average sidelobe power in the specified sidelobe region increases with the size of the sidelobe region as the nulls become spaced more widely apart. For a given sidelobe region the average sidelobe power in the specified sidelobe region decreases as the number of elements increases.

We conclude that if all except one degrees of freedom of the element weights are utilized to minimize pattern power in a specified sidelobe sector, with the remaining one degree of freedom used to satisfy a mainbeam constraint condition, then a pattern can be formed with low average sidelobe power in a broad sector. The larger the number of array elements, the lower will be the average power in the specified sidelobe sector. The price paid, however, for this sidelobe sector power reduction is a marked degradation of the remainder of the pattern.

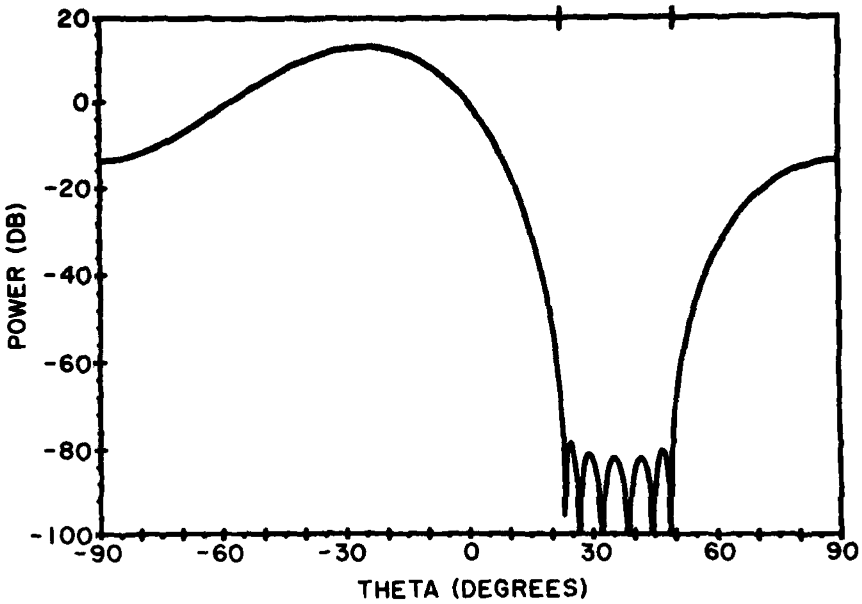

Figure 2a. Seven Element Pattern: Minimized Sidelobe Sector = [22.40°, 50°]

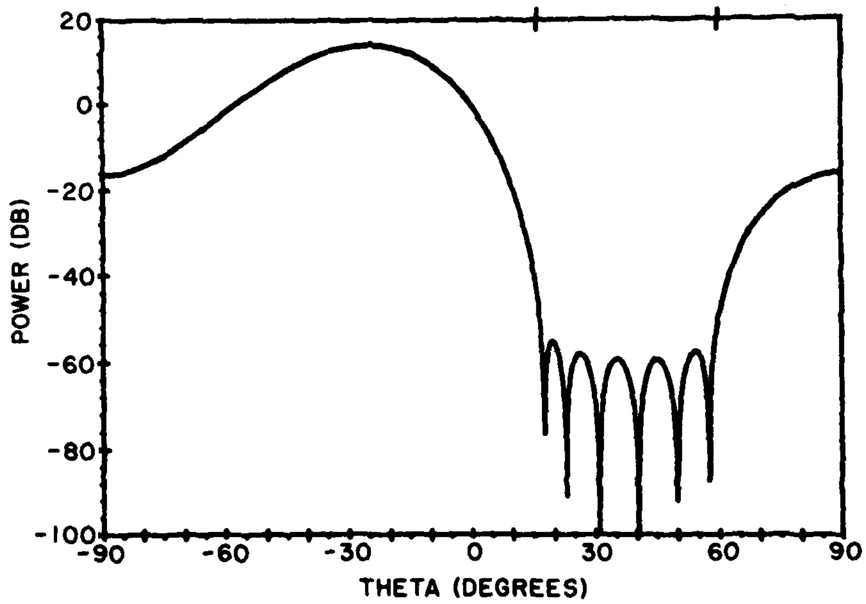

Figure 2b. Seven Element Pattern: Minimized Sidelobe Sector = [16.33°, 60°]

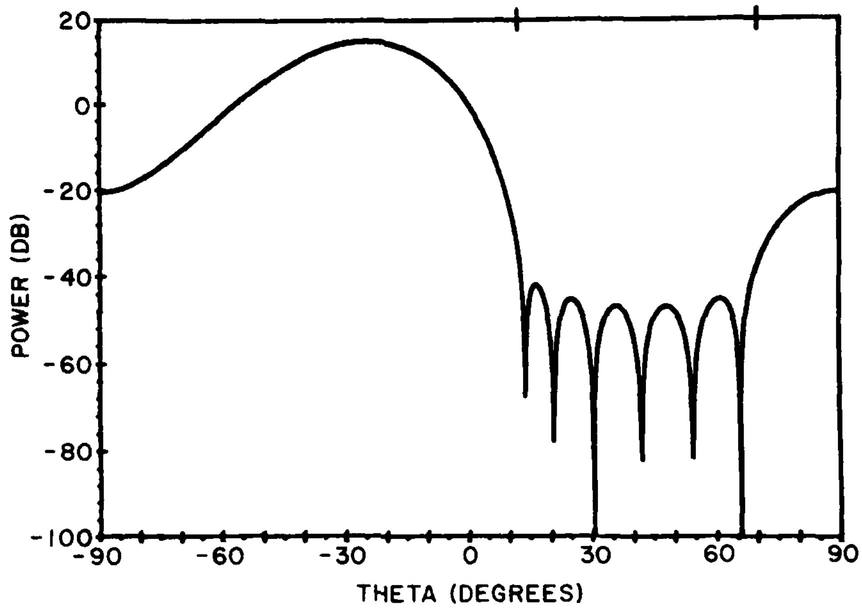

Figure 2c. Seven Element Pattern: Minimized Sidelobe Sector = [11.97°, 70°]

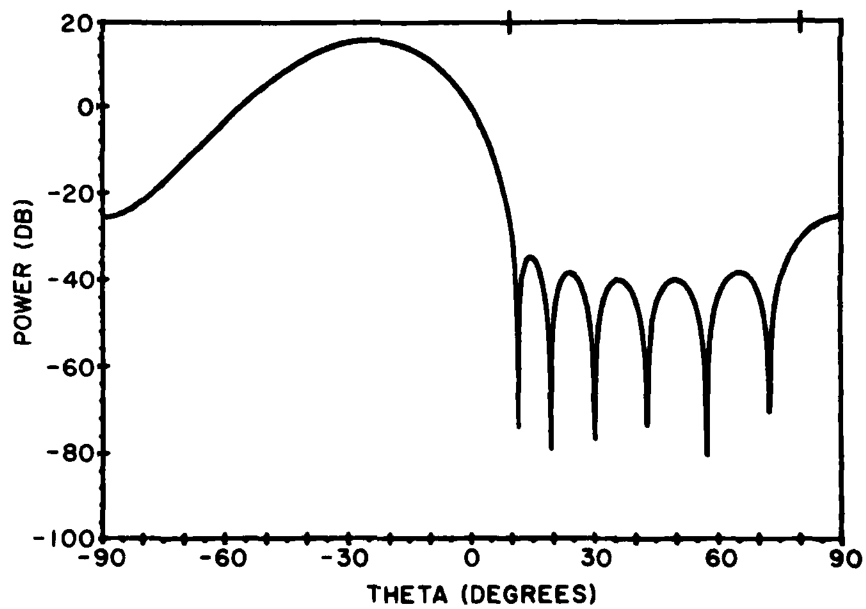

Figure 2d. Seven Element Pattern: Minimized Sidelobe Sector = [9.34°, 80°]

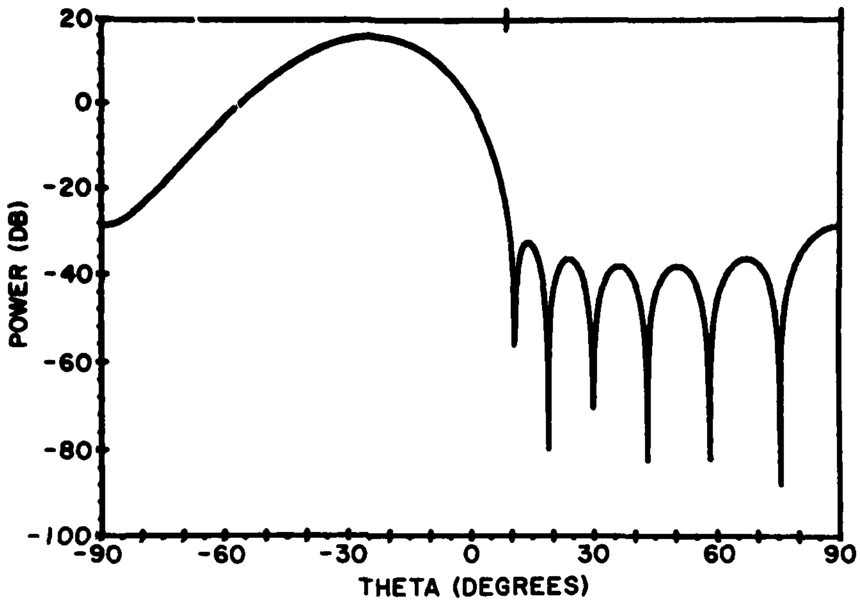

Figure 2e. Seven Element Pattern: Minimized Sidelobe Sector = [8.46°, 90°]

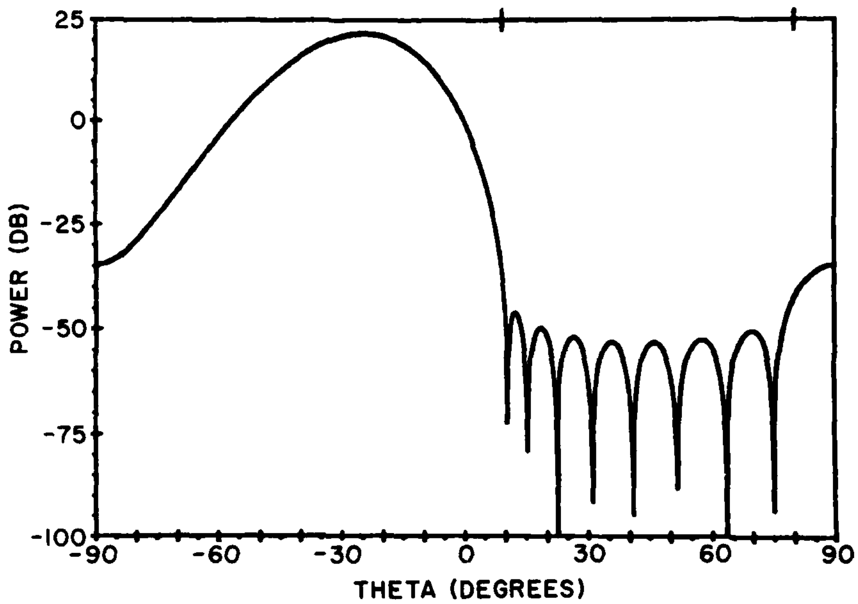

Figure 3a. Nine Element Pattern: Minimized Sidelobe Sector = [9.34°, 80°]

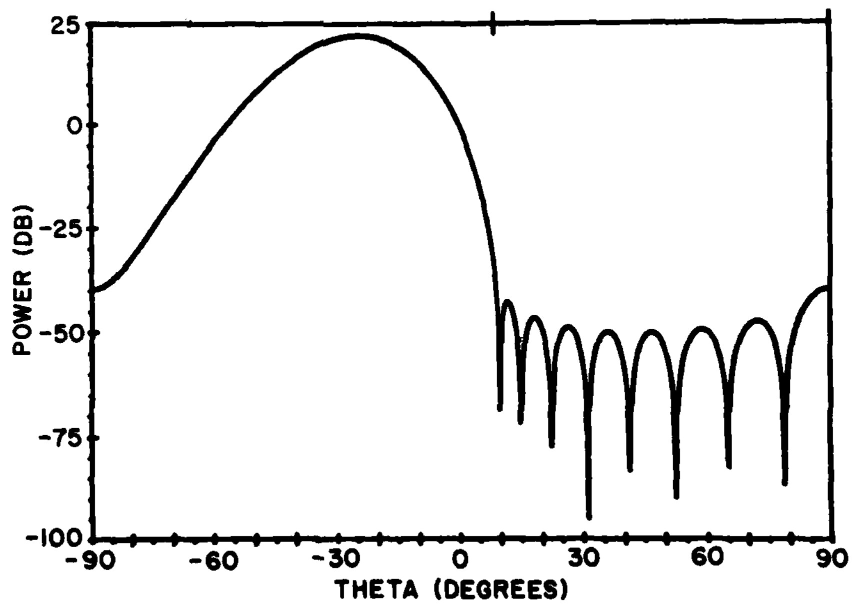

Figure 3b. Nine Element Pattern: Minimized Sidelobe Sector = [8.46°, 90°]

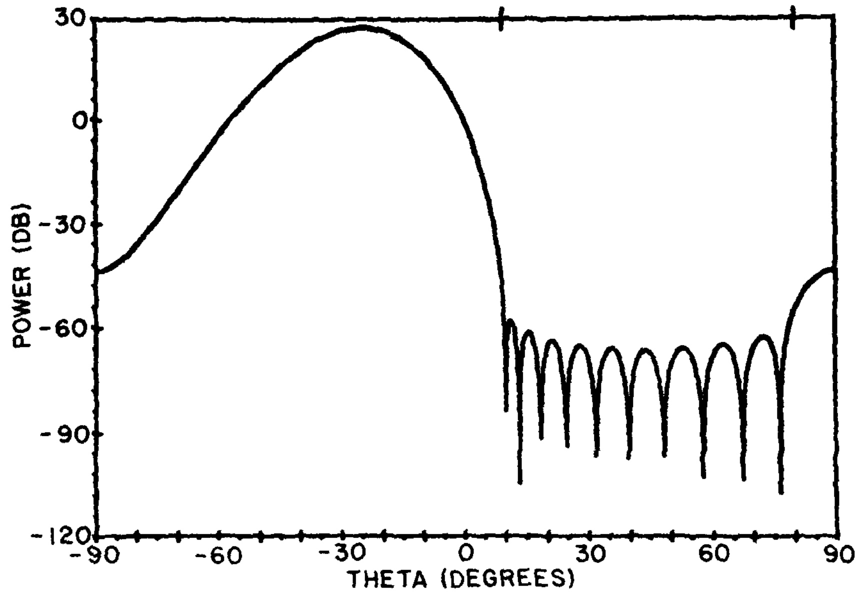

Figure 4a. Eleven Element Pattern: Minimized Sidelobe Sector = [9.34°, 80°]

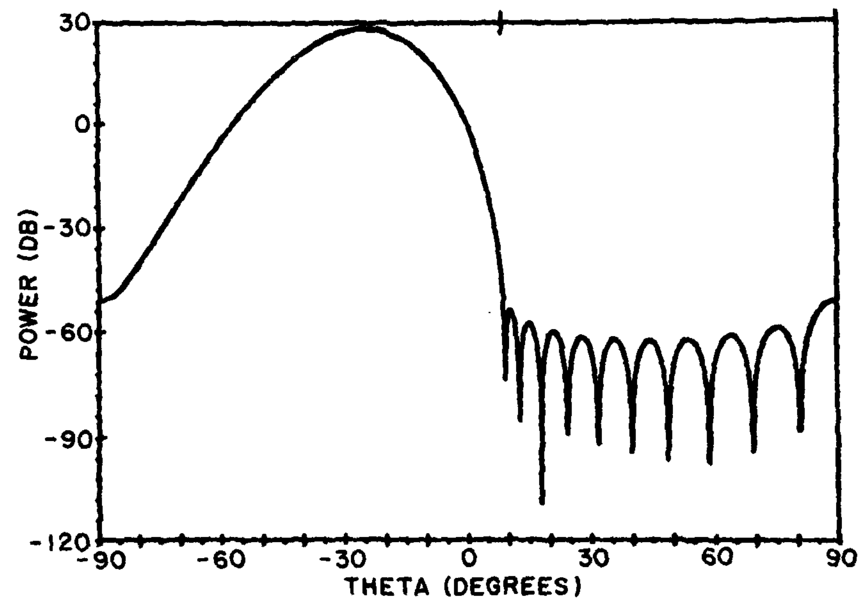

Figure 4b. Eleven Element Pattern: Minimized Sidelobe Sector = [8.46°, 90°]

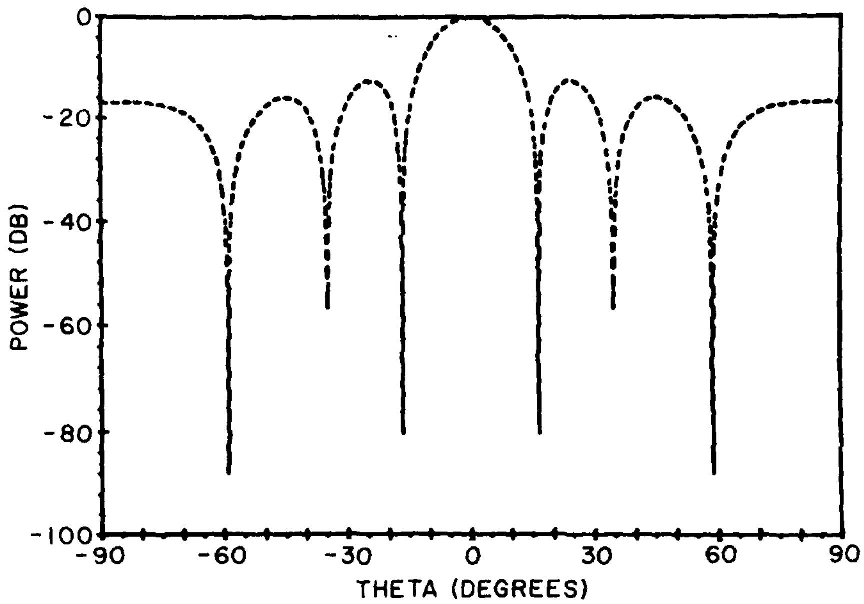

Figure 5a. Seven Element Pattern Uniform Weighting

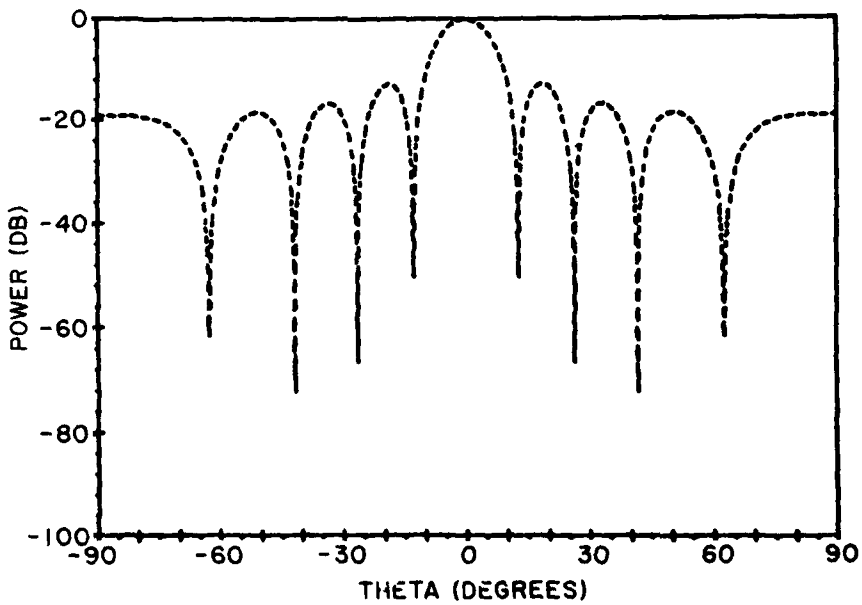

Figure 5b. Nine Element Pattern Uniform Weighting

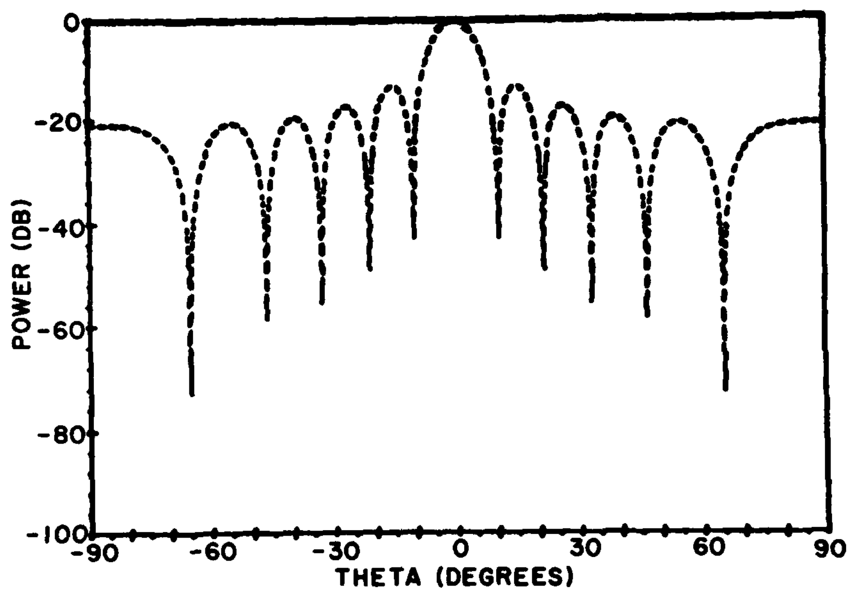

Figure 5c. Eleven Element Pattern Uniform Weighting

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